Exercise: Estimating the Mass of Jupiter
Difficulty: Medium

OBJECTIVE

The July / August observing notes for 2010 state that “Jupiter rises at dusk. The great planet is now starting its grand showing for the late Summer and Autumn”. Jupiter’s ideal position in the sky also presents a great opportunity for amateur astronomers to discover for themselves how scientists have been able to determine some of the important characteristics of our solar system.

In this simple practical experiment you will use observational data acquired over a number of evenings using your own telescope – and just a little basic mathematics - to estimate the mass of the planet Jupiter, the largest planet in our solar system.

EQUIPMENT

All you need to perform this experiment are:

- A telescope with enough magnification to show Jupiter as a small disk
- A pen and paper to record your observations
- An up to date star map to give you the location of Jupiter in the sky

SOME BACKGROUND – HOW DO YOU WEIGH A PLANET?

Kepler discovered the laws of planetary motion in 1609 by analyzing Tycho Brahe’s observations of planetary motion. Kepler’s three laws of planetary motion describe the orbits of planets about the Sun. Isaac Newton later generalised these laws to describe the motion of any two bodies orbiting their common centre of mass.

First Law:	The relative orbit of two bodies is a conic section with one of the objects at a focus.
Second Law:	The line connecting the two bodies sweeps out equal areas in equal times.
Third Law:	The product of the square of the period and the total mass of the system is proportional to the cube of the mean separation of the bodies

You can use Newton’s version of Kepler’s third law to determine the total mass within any system of orbiting bodies by using measurements just so long as you can determine the orbital period and the separation.

METHOD

This very simple method can be used to estimate the mass of Jupiter from your own observations of its moons' orbits:

1. Using your telescope, measure the positions of the four moons of Jupiter (Io, Europa, Ganymede and Callisto) at regular intervals.

2. Plot your observations for each moon on a graph (when the clouds permit)

3. Determine the period (T) and semi-major axis (a) for the orbit of each moon from your graphs.

4. Calculate the mass of Jupiter as the average of the four values for Jupiter's mass calculated from your individual observations.

Note that the principles behind this method are used by Astronomers to calculate the masses of the other planets in our solar system, the mass of the Sun, the masses of binary stars and even galaxies.

Step 1: Measuring the positions of Jupiter's four moons

To measure the positions, use the highest magnification that still allows all four moons to occupy the field of view with Jupiter centred in the field of view. Estimate the distance as the number of Jupiter diameters from the centre of the planet to the moon (or better still, use a reticule eyepiece if you have one).

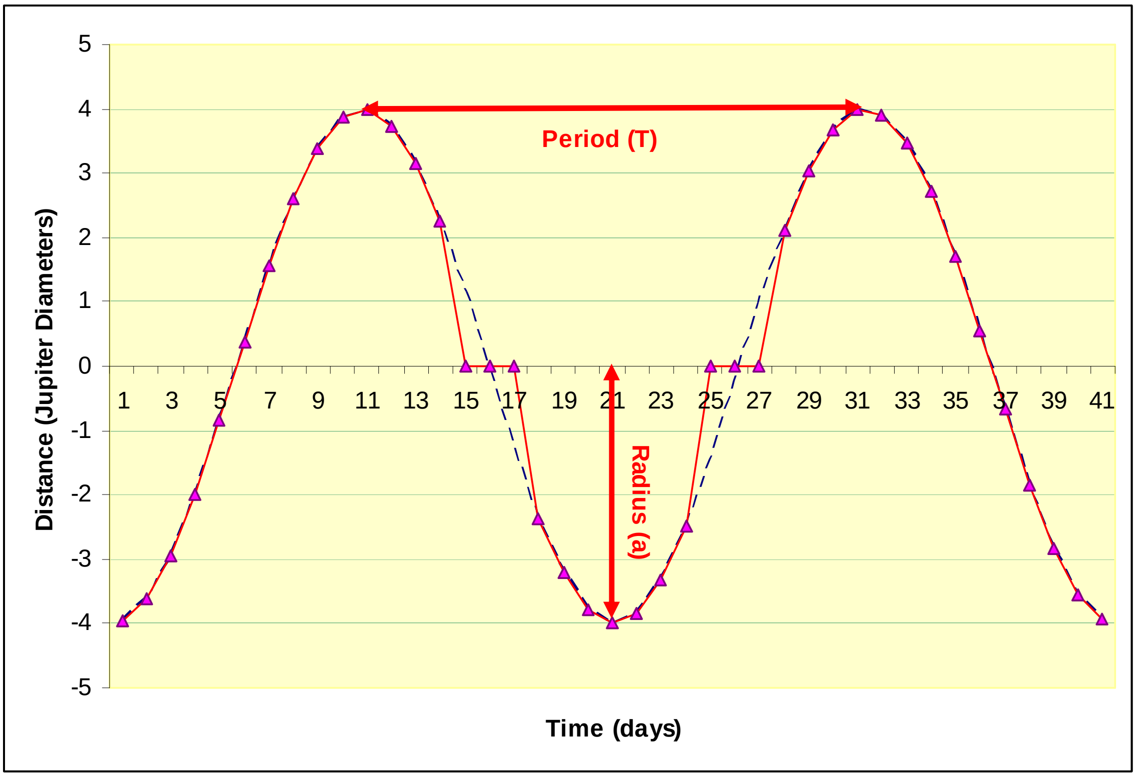
Herein lays your first challenge – how do you tell which moon is which? You might be able to work this out by looking at your data knowing that Io completes an orbit in about two days, Europa takes three and a half days Ganymede takes a little over seven days and Callisto has a period of over two weeks. Remember also that the radius of Io's orbit is much smaller than that of Callisto. Alternatively you can use ephemeris data published in an astronomy magazine to help you.

Record the date, time and distances. You can use the observation record sheet at the end of this document to record observations – print off as many copies of this sheet as you need to record your nightly observations. When you record the distances make the distance a positive value if the moon is to the West of Jupiter and a negative value if the moon is to the East of Jupiter (alternatively mark your measurements with an "E" or "W" as appropriate". You will also find that moons are not visible when their orbit takes them behind the planet (simply record these distances as zero).

Observations every 24 hours (if the clouds permit) should allow you to create a rough curve for Callisto but more frequent measurements will be required for the other moons. The more measurements you make the more smooth your curve will be.

Step 2: Plot your data

Plot a graph of distance (Y axis) against time in days (X Axis). Remember to plot fractions of days correctly (i.e. 1800hrs is 0.75 of a day). The graph below shows a data plot for an imaginary moon. With gaps in your data don't expect to see anything quite this perfect. If you have too much uncertainty then take more frequent observations until you have collected enough good quality data.



Step 3: Determine the period and semi-major axis

Sketch a smooth curve which best represents the data and taking in as many points on your graph as possible. The curve should be a sine curve. Consider rechecking points which look very different from their expected position. Don't worry about the zero values when the moon disappeared behind Jupiter, just plot the zeros on the graph as shown above.

The period T and the radius a can be read from your graph directly. In this made-up example:

- Period (T) = 10 days (8.64 x 10^5 seconds).
- Radius (a) = 4 Jupiter diameters (approx 5.72x10^8 metres).

Step 4: Calculate the mass of Jupiter

This is where the maths comes in, but thankfully it is all basic stuff. To calculate the mass of Jupiter you simply use Newton's version of Kepler's Third Law.

$$T^2 = \frac{4\pi^2}{G} \cdot \frac{a^3}{M + m}$$

Where ***M*** is the mass of Jupiter and ***m*** is the mass of the moon and ***G*** is the universal gravitational constant (6.67 x 10^{-11} m^3 / kg-sec^2). Because the mass of the moon (***m***) is so small compared to the mass of Jupiter (***M***) we are able to remove m from the equation. Rearranging we get:

$$M = \frac{4\pi^2}{G} \cdot \frac{a^3}{T^2}$$

Simply plug in your values for ***a*** and ***T*** into this formula to calculate the mass of Jupiter using your data for the four moons (remembering to convert the radius ***a*** into metres and the period ***T*** into seconds) then take the average value.

ANALYSIS OF YOUR RESULTS

Look up the mass of Jupiter (look it up on the web, try using the Wolfram Alpha search engine). Are your results consistent with the currently accepted value? If not, why do you think this is?

Lookup details of the period and radius of the Moon's orbit and try calculate the mass of the Earth. Is the margin of error any more or less than your calculation for Jupiter. Why do you think this is?

If you try this experiment for yourself let us know how you got on and how well your result compared with the accepted figure. You can use the comment form at the bottom of this post to leave your answer.

If you are feeling adventurous try this approach to calculate the mass of Saturn; this is a little harder because the moons are fainter and unlike Jupiter the moons of Saturn are not conveniently in a line. You will need to take a little more care calculating the radius of the orbits.

Observing Log Sheet #:_____

Observation details		Distance from Centre of Jupiter (in Jupiter Diameters)			
Date	Time	Io	Europa	Ganymede	Callisto