Exercise: Understanding Positional Astronomy

Part 2 –Celestial Co-ordinates

Difficulty: Intermediate

Objectives

In Part 1 you learned about Celestial Sphere and how the stars appear to move across the night sky. In this second part on Positional Astronomy you will learn how astronomers describe the position of a celestial body on the celestial sphere.

In fact astronomers use several different co-ordinate systems to describe the apparent positions of stars on the Celestial Sphere, the most common of these being the Alt-Azimuth System and the Equatorial Coordinate System. These are described below along with three exercises in positional astronomy.

Astronomers sometimes find it necessary to convert the co-ordinates from one system to another. For example, if you are lucky enough to own a computerised 'GOTO' telescope have you ever wondered how the telescope knows where to find objects? Most of these will perform a conversion from equatorial co-ordinates held in its database to alt-azimuth co-ordinates to point the telescope at objects in the night sky. In one of the exercises you will learn how to convert equatorial co-ordinates to alt-azimuth co-ordinates with just the aid of a calculator.

Equipment

To complete the exercises below you will need:

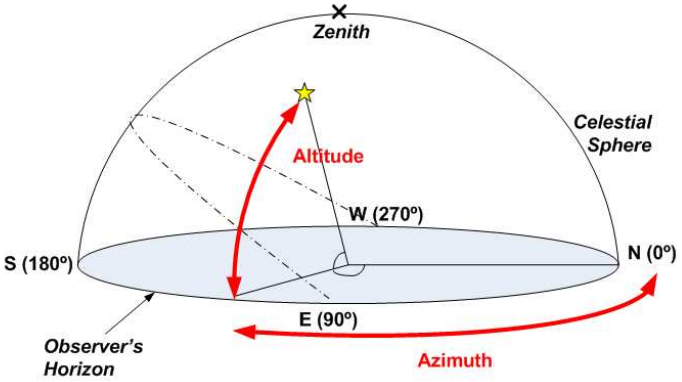
- A star map
- A long stick or bamboo cane
- A clock or wristwatch
- Pen and paper to note your observations
- A scientific calculator (with trigonometric functions)
- A compass

Describing Positions on the Celestial Sphere

Alt-Azimuth Coordinates

In the Alt-Azimuth system the positions of celestial bodies in the sky are given relative to your particular location on the earth. In part 1 of Understanding positional Astronomy we learned that the Zenith is a point directly above you and the Meridian is a great circle that passes through the Zenith, due North and due South points on the horizon. Stars will reach their highest altitude as they cross the Meridian. To describe the position of a celestial body using the alt-azimuth system we think of a great circle passing through the star's position and the Zenith. We can use this great circle to specify two coordinates to locate the celestial body.

- ***Azimuth*** is the angle between due North on the horizon and the intersection of the horizon with the great circle through the celestial body where North is 0º, East is 90º, South is 180 º and West is 100º.
- ***Altitude*** is the angle of of elevation between the celestial body and the horizon – it is measured from 0 º at the horizon to 90 º at the Zenith.



Measuring altitude can be difficult because your visible horizon may be different from the true horizon (if for example, you live in a shallow valley or near mountains). You might find it easier to judge the angle from the Zenith because the "straight up" direction is easier to determine (the azimuth is 90º minus the angle from the Zenith).

The problem with this system is that a celestial body's azimuth and altitude will change during the night because the positions are measured from a fixed reference points with respect to the observer – North (azimuth) and the horizon (altitude).

Also, someone else standing at a different location (a different latitude and longitude) would see a different position in the sky and therefore would measure a different altitude and azimuth. Imagine telling a friend on the other side of the Atlantic about a wonderful galaxy or nebula you saw last night. How would you tell him where it is in the sky?

Equatorial Co-ordinates

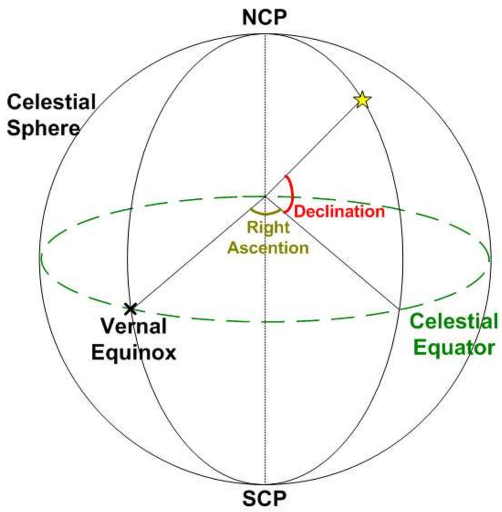
The equatorial system avoids the uncertainty of the Alt-Azimuth system by measuring positions are relative to a known point on the celestial equator (the Vernal equinox). In this way the co-ordinates don't change with respect to an observer's location – the information telling an astronomer where the object is is the same for every observer regardless of date, time or location.

In the equatorial co-ordinate system we describe a celestial body's position by their Right *Ascension* (RA) and *Declination* (Dec).

First of all, to properly understand the Equatorial System it is important to understand the concept of an 'hour circle'. Any great circle projected onto the on the celestial sphere which passes through both the North and South celestial poles is called an 'hour circle'.

Right Ascention and Declination are in effect a projection of the Earth's latitude and longitude onto the Celestial Sphere:

- ***Right Ascension (RA)*** is the angular distance of a celestial body measured eastwards along the celestial equator from the Vernal Equinox to the hour circle passing through the celestial body. This is usually measured in units of time (hours, minutes and seconds) with 24 hours being equivalent to 360°.
- ***Declination (Dec)*** is the angular measure north (positive) or south (negative) of a celestial body from the celestial equator. Declination is measured in degrees from 0º at the celestial equator to 90º at the celestial pole (negative values are used for the bodies south of the celestial equator).



Local Hour Angle

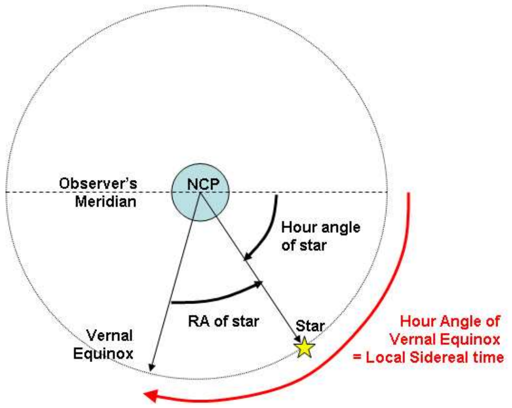
The local hour angle of a celestial body is the angle (measured Westwards) from the observer's meridian to the hour circle passing through the celestial body. The local Hour Angle can be used as a measure of the time since a celestial body crossed the observer's meridian. For example, a star with an HA of 4 hours crossed the observer's meridian 4 hours ago. Similarly a celestial body with an HA of 16h crossed the observer's meridian 16 hours ago (it will do so again in six hours). Note that the HA is measured with respect to the observer's meridian which is a reference point relative to the observer and is therefore different for observers at different locations on the Earth. The hour angle of a celestial body will change by one hour for every 15º difference in longitude and for a fixed observer, the hour angle of a celestial body will change by one hour every hour with the apparent movement of the stars (remember that your meridian is fixed).

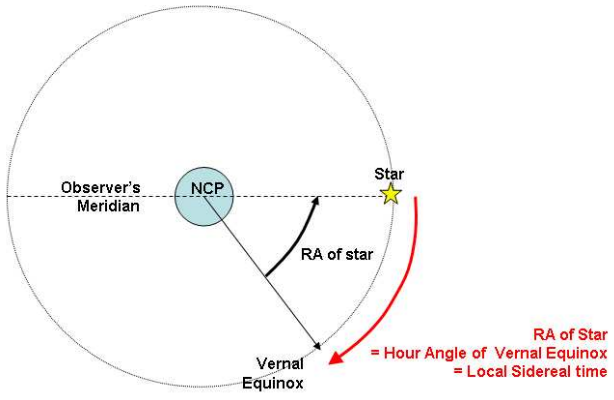
Sidereal Time

The every-day time we use is mean solar time and it is measured with respect to the Earth's rotation and the position of the Sun - where 12 o'clock mid-day is when the Sun crosses the meridian. Another measurement of time that is important to astronomers is Sidereal time which is measured with respect to the stars. The sidereal day is the time between successive transits of the Vernal Equinox through the observer's meridian. In solar time 1 day is 24 hours but the Sidereal day is slightly shorter. A Sidereal Day equals 23h 56m 4 seconds of mean Solar Time. The sidereal day begins at 0h when the vernal equinox across the observer's meridian and because a solar day is longer than a sidereal day, the Sun appears to move backwards against the stars by about 4m or 1° each day.

Why is the sidereal day shorter? It is because as the Earth rotates it is also orbiting the sun. Due to the orbit the stars return to their original positions about 4 minutes earlier each day.

Your local sidereal time is given by the RA of a celestial object that is crossing the meridian. The measurement is the same as the hour angle of the Vernal Equinox (because RA is measured from the Vernal Equinox).





The Greenwich sidereal time is the sidereal time as measured at the longitude of the old Royal Observatory at Greenwich which lies on the Greenwich meridian (0º longitude). This means that your local sidereal time is equal to the Greenwich sidereal time minus your longitude.

Exercises

Exercise 1 - Finding the Altitude and Azimuth coordinates of a star

In this simple exercise you will use some basic equipment (a compass and protractor) to measure the Altitude and Azimuth of a star.

- Use a star map to find the coordinates of a bright star that will be visible easily from your location then use the constellations to identify the same star in the night sky. Try to chose a star that is near to the horizon as this will help you measure the beading more accurately.
- Using the protractor and weighted string you used in Part 1 to measure the altitude of the star you have identified.
- Note down the date and accurate time of your observation.
- Now, using your compass work out the bearing to your star in degrees; convert this bearing into hours, minutes and seconds (remember, 1 hour = 15º). This is the star's azimuth.

If you have access to the Internet (or an ephemeris or planetarium program on your PC) lookup the altitude and azimuth of your star – but remember, you will have to tell the software, your exact location, the exact date and exact time of your observation. How does your result compare? How do you account for any difference? (a hint, did you remember to take account of daylight saving? Did you measure the bearing as true North or magnetic North?)

Exercise 2 – Finding you local Sidereal time

In this exercise you will estimate your Local Sidereal Time with just a simple observation.

- From a good vantage point with a clear view to the South mark your exact location with a chalk mark or a pebble at your feet.
- Without changing your position (keep your feet at the mark) find the pole star and draw an imaginary line from the pole star through the zenith and down towards due South. This is your meridian line. Mark this line by getting a friend to push a long stick (e.g. a bamboo cane) into the ground. This stick will be your reference point for the meridian.
- Now identify a star near to and to the East of the meridian – the lowest star you can find will give you best results. Keep observing this star until it is right on the meridian (if you have a long wait you can always go inside for a cup of tea – so long as when you return you return to exactly the same spot).
- When the star is on your Meridian (use the stick or bamboo cane as a guide) take a note of the time (this is the civilian or solar time).
- Using your star map look up the Right Ascension of the star – this is the local Sidereal time.

To validate your result find a Sidereal Time calculator on the Internet (such as http://www.jgiesen.de/astro/astroJS/siderealClock/). Be careful to enter your correct longitude and the time recorded when the star crossed your meridian. How accurate was your observation? Can you explain any errors - did someone move your stick whilst you were having that cup of tea or is there some other explanation?

Exercise 3 - Converting from Equatorial to Alt-Azimuth co-ordinates

In this exercise you will convert Equatorial co-ordinates of an object to Alt-Azimuth co-ordinates with just the aid of a calculator. This requires a little bit of mathematics.

Assuming that the observer is at latitude ϕ, then for a star with RA θ, declination δ, then we can convert from equatorial co-ordinates to alt-azimuth co-ordinates (altitude α and Azimuth A) using the following formulae:

$$h = LST - \theta$$

$$\sin(a) = \sin(\delta)\times\sin(\phi) + \cos(\delta)\times\cos(\phi)\times\cos(h)$$

$$\cos(A) = \frac{\sin(\delta) - \sin(\phi)\times\sin(a)}{\cos(\phi)\times\cos(a)}$$

These equations look complicated but we have broken-down the calculation into easy steps and to help you further we have provided a worked example. A worksheet is also provided at the end of this document.

Try to follow the steps in the worked example. This takes the star Alpha Lyrae (Vega) as an example and assumes that the observer is observing from a latitude of 53º 27' at a little before 1am (time 00:55) on 7 August 2010. The equatorial co-ordinates of Vega are RA: 18h 37m 19.97s and Dec +38° 47' 49.3". The local sidereal time (L ST) is 21h 46m 44s.

Now try it by yourself:

- Use your star map to find an star visible easily at your observing location and use the star map find its Right Ascension and Declination.
- Using the method in Example 3 work out your Local Sidereal Time
- Using the worksheet, work out its Altitude and Azimuth at your observing location for a Local Sidereal Time, say, exactly one hour ahead.
- Using your calculated altitude and azimuth look to where you think the star should be:
    - Use a compass to estimate the direction based on your azimuth (remember 15º = 1 hour)
    - Use the protractor in exercise 1 above to observe in the region defined by your calculated altitude.

How close is your star to the calculated position?

Exercise 3 - Worked Example

	Step	Example
1	Convert LST into decimal hours	= 21 + 46 /60 + 44/3600 = 21.778889
2	Convert RA into decimal hours	= 18 + 37/60 + 19.97/3600 = 18.622214
3	Calculate h by subtracting result 1 from result 2	= 3.156675
4	Convert h from hours to degrees (1 hr = 15 degrees)	= 3.156675 x 15 = 47.350125
5	Convert declination (δ) into decimal degrees	= 38 + 47/60 + 49.3/3600 = 38.797027
6	Convert latitude (ϕ) to decimal degrees	= 53 + 27/60 = 53.45
Calculate Altitude		
7	Calculate $\sin(\delta)\times\sin(\phi)$	sin(38.797027) = 0.626563 sin(53.45) = 0.8033374 $\sin(\delta)\times\sin(\phi)$ =0.503341
8	Calculate $\cos(\delta)\times\cos(\phi)\times\cos(h)$	cos(38.797027) = 0.779370 cos(53.45) = 0.595524 cos(47.350125) = 0.677516 $\cos(\delta)\times\cos(\phi)\times\cos(h)$ =.0.314458
9	Calculate α as result 7 - result 8	0.817799
10	Take the inverse sine to give the altitude	54.865068 (54º 51’ 54”)
Calculate Azimuth		
11	Calculate $\sin(\delta)-\sin(\phi)\times\sin(a)$	sin(38.797027) = 0.626563 sin(53.45) = 0.8033374 sin(54.865068) = 0.817799 $\sin(\delta)-\sin(\phi)\times\sin(a)$ = -0.030405
12	Calculate $\cos(\phi)\times\cos(a)$	cos(53.45) = 0.595524 cos(54.865068) = 0.575504 $\cos(\phi)\times\cos(a)$ = 0.342726
	Calculate $\dfrac{\sin(\delta)-\sin(\phi)\times\sin(a)}{\cos(\phi)\times\cos(a)}$	= -0.088716
13	Take the inverse cosine to give the altitude A	= 95.089744
14	If $\sin(\delta)>0$ then $A = 360 - A'$ else If $\sin(\delta)\le 0$ then $A = A'$	A = 360 - 95.089744 A = 264.910256 degrees (264 º 54’ 36.9”)
15	Calculated result: Altitude +54º 51’ 54”, Azimuth 264 º 54’ 36.9” Actual: Altitude +54°52' 36", Azimuth 264°54' 39 " (Source, Skymap) Note there is a small error of approximately 42” – this is probably rounding error.	

Exercise 3 - Worksheet

	Step	Example
1	Convert LST into decimal hours	= ___ + __ /60 + ____/3600 = ___________
2	Convert RA into decimal hours	= ___ + __/60 + ____/3600 = ___________
3	Calculate h by subtracting result 1 from result 2	= ___________
4	Convert h from hours to degrees (1 hr = 15 degrees)	= ___________x 15 = ___________
5	Convert declination (δ) into decimal degrees	= ___ +__/60 + ____/3600 = ___________
6	Convert latitude (ϕ) to decimal degrees	= ___ + __/60 + ____/3600 = ___________
Calculate Altitude		
7	Calculate $\sin(\delta)\times\sin(\phi)$	sin(___________) = ___________ sin(___________) = ___________ $\sin(\delta)\times\sin(\phi)$ = ___________
8	Calculate $\cos(\delta)\times\cos(\phi)\times\cos(h)$	cos(___________) = ___________ cos(___________) = ___________ cos(___________) = ___________ $\cos(\delta)\times\cos(\phi)\times\cos(h)$ = ___________
9	Calculate α as result 7 - result 8	___________
10	Take the inverse sine to give the altitude	___________ (__º __’ ____”)
Calculate Azimuth		
11	Calculate $\sin(\delta)-\sin(\phi)\times\sin(a)$	sin(___________) = ___________ sin(___________) = ___________ sin(___________) = ___________ $\sin(\delta)-\sin(\phi)\times\sin(a)$ = ___________
12	Calculate $\cos(\phi)\times\cos(a)$	cos(___________) = ___________ cos(___________) = ___________ $\cos(\phi)\times\cos(a)$ = ___________
	Calculate $\dfrac{\sin(\delta)-\sin(\phi)\times\sin(a)}{\cos(\phi)\times\cos(a)}$	= ___________
13	Take the inverse cosine to give the altitude A’	= ___________
14	If $\sin(\delta)>0$ then $A=360-A'$ else If $\sin(\delta)\le 0$ then $A=A'$	A = 360 - ___________ A = ___________ degrees (___ º __’ ____”)
15	Calculated result: Altitude ___º __’ ____”, Azimuth ___º __’ ____”, Actual: Altitude ___º __’ ____”, Azimuth =___º __’ ____” (Source _________________)	